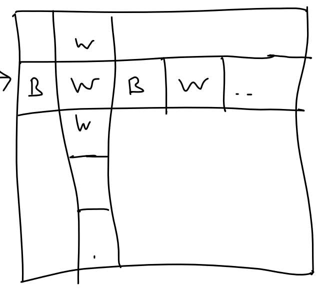
Problem Solving Strategies 5

22 December 2023 18:30

Q) There is a 8×8 chessboard with usual black and white colouring. We can repaint all squares of a row or coloumn in each step. Can we reach one black square?



Initially 4b 4w ⇓ 4w 4b

Ans:— Suppose in a row or column there are x black squares and $8-x$ white squares.

After 1 step on that row or column we get, x white and $8-x$ black.

So change in black squares $= |x - (8-x)| = |2x-8|$ → this is the invariant

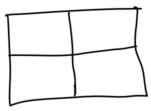
→ $2x$ is even, 8 is even $\Rightarrow |2x-8|$ is even

Initially there are $\frac{64}{2} = 32$ blacks and change is even so parity of blacks will also be even

So not possible.

Q) There is a 8×8 chessboard with usual black and white colouring. We can repaint all squares of a 2×2 square in each step. Can we reach one black square?

Ans:— Change in black $= |2x-4|$ is even is invariant ⇐



x blacks, $4-x$ white

HomeWork

Q) There are w white, b blacks and r red chips on a table. In each step we can take two chips of different colours and replace them by a chip of the third colour. If just one chip remain at the end its colour will not depend on the evolution of the game. On what condition this will happen?

HomeWork

Q) Each term in a sequence $\overset{a_1}{1}, \overset{a_2}{0}, \overset{a_3}{1}, \overset{a_4}{0}, \overset{a_5}{1}, \overset{a_6}{0}, \ldots$ starting with the seventh is the sum ~~of the~~ last 6 terms mod 10

Q) Each term in a sequence [illegible] seventh is the sum of the last 6 terms mod 10 (eg. $a_7 = 3, a_8 = 5, a_9 = 0, \ldots$). Prove that the sequence, $\ldots 0, 1, 0, 1, 0, 1, \ldots$ never occurs.

Hint :- $x_1 a_1 + x_2 a_2 + x_3 a_3 + x_4 a_4 + x_5 a_5 + x_6 a_6$ take it such a way such that this gets to an invariant.